The High Resolution Scene Brightness or BRDF Task for the CSIRO EOC

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1. Introduction

This document outlines a suggested framework for the high resolution Scene Brightness or BRDF Task for the CSIRO EOC. At this time there is a well defined pair of objectives that a Task can follow and achieve. I believe this can all be done within a year.

The two primary objectives are:

1. **Objective 1**: To develop guidelines for best practice outputs (including prototype software) for high resolution Aerial Photography, Video, Digital Camera and Airborne Scanner data in terms of scene brightness.

2. **Objective 2**: To advance knowledge concerning the consistency of land cover BRDF at the high spatial resolution image scale

Objective 1. Includes further development of the referencing image method, its comparison and potential interactions with modelling and kernel approaches and code establishment for the cases. It also includes the development of field protocols for testing 'spectral' (but not necessarily hyperspectral) integrity and consistency of the outcomes of the methods. The result should be a general suite of tools that people can apply to a wide range of cases and a documented means of deciding what choices of options are available for specific applications.

Objective 2 is complementary to the AVHRR/GMS/ATSR scale work going on among the AVHRR Task scientists primarily at Atmospheric Research. The results of comparing different models based on the frames or the fourier method residuals will provide valuable answers at the high resolution end.

As a result of the project, people using these data types should have access to methods and code that realise the methods to normalise and standardise their high-res data consisting of many frames. The software effort will need to be shared out and the design for that component of the Task should be part of the proposal.

The final source code will need to be maintained by the groups responsible and binary executables be the primary means of distribution. The software should take account of that being developed for the AVHRR scale work and (where possible) be fully compatible or identical.

Overall, we need to be in a position where a range of high resolution airborne data processing tools are available and being put to use. In the end, the real "market" is in the quality products and maybe not the kind of software being considered here. Without effective and defendable solutions to the issues of geometric and spectral mosaicking of airborne data there may be no users.

2. General framework for the development:

2.1 "Pages" and the duality of geometric and radiometric models.

This formulation uses the idea of a "Page" which is usually an image or a map or the section of the world that is being mapped.

Among the Pages there is one nominated called the "Base Page" or "Reference Page". The total number of Pages is assumed to be N_{page}+1. That is, N_{page} Pages and one Base Page. Each page has a geometric model and a radiometric model mapping the data in the page to the base page of the form:

Geometric Model (G_j):

$$x_b = G_j(x_j, \mu_j)$$

where:

x_j is the position of pixel or object in Page j
μ_j is a set of parameters for the model in Page j and
x_b is the mapped position in the Base Page corresponding to x_j

Radiometric Model (R_j):

$$\rho_b(x_b) = R_j(\rho_j(x_j), \theta_j)$$

where:

ρ_j is a vector of channel data;
θ_j is a set of parameters for the radiometric model in Page j; and
$\rho_b$is the mapped vector of channel data (maybe not all channels) in the Base Page.

The assumption is that many of the pages overlap and that equivalent geometric and radiometric data objects can be located in the overlaps. These points are Tie Points. They can be patches but here it assumed the data are assigned to the centroid and the radiometric data are the average over the patch.

Formally, if there are N_{jb} points common to Page j and the Base Page and there are N_{jm} points identified in the overlap between Page j and Page m then geometric modelling of the "Target Page" model (Page j) is seeking to adjust any parameters of G_j (μ_j) and the other Pages (μ_l, l=1,N_{page} $l \neq j$) such that:

$$G_j(\mu_j, x_{jk}) \approx x_{bk} \qquad for\ k = 1, N_{jb}$$
$$G_j(\mu_j, x_{jl}) \approx G_m(\mu_m, x_{ml}) \qquad for\ l = 1, N_{jm}$$

The parameters are not, of course, only defined by the above pages and immediate relations. All pages "connected" with the pages of interest are involved. By "connected" is meant the transitive closure of the graph containing Page j that is formed by defining an undirected link between two pages when they overlap (and have Tie points) and then "close" the graph such that if A and B overlap and C overlaps B (but not A) then A and C overlap with "distance" 2.

Hence, all of the models and overlap constraints will be involved simultaneously in the solution of this system. Only the choice of Base Page and Target Page will change. In some cases, models for all the Pages as "targets" will be available after one solution for the mapping into the base Page but for the optimal mapping of the Base Page into one of the Pages there will be one solution for each Page.

Software involved here must carefully define and analyse the connection matrix as it defines how well posed the problem of determining parameters is.

The full system mapping the Pages into the Base Page runs as:

$$
\begin{aligned}
&G_j(\mu_j, x_{jk}) \approx x_{bk} && \mathit{for}\ \ j = 1, N_{page}, k = 1, N_{jb} \\
&G_j(\mu_j, x_{jl}) \approx G_m(\mu_m, x_{ml}) && \forall\ \mathit{unique}\ (x_{jl}, x_{ml}), l = 1, N_{Tie}
\end{aligned}
$$

where N_{tie} is the total number of different Tie points in the connected set of Pages.

The parameters are fitted as in any such "inversion" problem minimising some combination of functionals that measure goodness of fit, regularisation ("smoothness") and complexity. An example of how this works with a linear model is shown in the Appendix outlining Mosmod and illustrates how the connections and mappings are organised in that case.

The dual problem to this is Radiometric modelling:

$$
\begin{aligned}
&R_j(\rho_{jk}(x_{jk}), \theta_j) \approx \rho_{bk}(x_{bk}) && \mathit{for}\ \ j = 1, N_{page}, k = 1, M_{jb} \\
&R_j(\rho_{jl}(x_{jl}), \theta_j) \approx R_m(\rho_{ml}(x_{ml}), \theta_m) && \forall\ \mathit{unique}\ (x_{jl}, x_{ml}), l = 1, M_{Tie}
\end{aligned}
$$

where:

> M_{Tie} is the number of unique Tie points where there are valid radiometric associations
> M_{jb} is the number of assigned values common to Page j and the Base Page.

The values assigned in the Base Page in this case are sometimes called Pseudo-Invariant Features or PIFs. They are the radiometric equivalent of GCPs.

2.2 Example 1 - Mosaicking of Image Pages

The first example is mosaicking image pages that have geometric models defined by a polynomial, spline or other linear function. This would be the case with most satellite data or airborne data it the Pages were frames from which the major distortions had been taken.

In this case, the Base Page is normally the map and the assignments of points to the Base Page are the GCPs. In this formulation, the issue is to both fit the GCPs in each frame or Page and also match the overlapping frames to the same degree of accuracy.

If the goodness of fit criterion is least squares then the solution is given in Appendix 1 where the Mosmod program is described. The advantage here is that, just like “Bundle Adjustment” the GCPs that you have can pass control into adjacent frames.

This formulation also fits the case of time series data where the mosaic is a set of frames at different times that are closely registered. Normally, each image is registered to a base page separately. However, forming Tie points (by automatic correlation methods if possible) it is possible using the complete formulation to simultaneously register all the images pairwise to each other. This is an advantage for time series products where ties between frames is often more crucial than accurate location in a map.

How well posed this problem is depends on the adjacency matrix structure and spread of control and Tie points.

The same formulation can be used to effect spectral balancing and create “seamless” mosaics or time series in which specific patches that are assumed not to change are preserved. The linear case (which includes kernels) is identical to Mosmod.

One possible way of combining the geometric and spectral cases is to triangulate the data in the overlaps and reduce differences between the mean radiances in the triangles. With automatic control point determination this is quite feasible to force “seamless” balancing.

2.3 Example 2. - The referencing Method

The referencing method for radiometric balancing is a way to overcome fitting to real “trends” in a frame of data which is not a “scene brightness” variation. This is done by assuming that the reference image (eg SPOT or Landsat TM) includes the main variations in the data but not the BDRF that occurs in the individual frames of the Pages (which could be DMSV or DATM or Aerial Photographs).

In the referencing image method, the Base Page is the Reference Image (eg a Landsat image) and the radiometric model for Page j is:

$$I_b = H(t_j) * I_j + (I - H(t_j)) * I_R$$

where:

I_j is the j’th Page or Frame
I_R is the reference image
$H(t_j)$ is a high pass filter with cut-off tj in Page j

The assumption is that the individual frame BRDF effects are low frequency components and that low frequency components of the frame which are “real” data can be provided from the reference image. This is not a problem if the main changes being monitored are spatial high frequency effects. There is (in effect) a “geostatistical” hypothesis here that low spatial frequency entities are also low temporal frequency entities but no more of this.

In the current method, the threshold is chosen separately for each Page. However, in the formulation presented here it would be possible to choose the thresholds to ensure small residuals in the overlaps and agreement with field measured PIFs but also to make tj as small as possible overall. The reason for this is that if tj=0 then H=I and the result is just the original image (possibly scaled so that the region near the centre is “like” the reference image) while if the tj are at the Nyquist then the result is the Reference image so there is no error in the overlap areas but also no data from the new Pages!

There are many formulations that would help in this case to form overall optimum selections of thresholds. For example, minimise the maximum tj such that the largest error in the overlapping areas is less than a given tolerance. One objective of the Task would be to test such options.

The referencing method can also be combined with the Kernel approach by assuming that:

$$I_b = I_R + \delta I_j$$
$$or$$
$$I_b = I_R \times \delta I_j$$

where the perturbation is the Kernel model and can have different parameters in each frame. The advantage of this is that when the perturbations are fitted then they can be used to correct the Pages without using the reference image. An added advantage is that these models are now available for checking out BRDF typology.

As before, the process of determining the parameters can be a single simultaneous modelling exercise which ensures the differences in selected overlapping areas are small and fit to any known data (such as reflectances of PIFs) is good as well.

2.4 Example 3 - Atmospheric Correction and non-linear models

This approach may also be used to atmospherically correct frames or images and ensure the effects in overlapping areas are the same. This can be combined with a Kernel model for the angular variations as well. In this case the model are non-linear but may be simplified by linearisation.

Linearisation may also be used for complex geometric models such as those based on aircraft attitude series. Assuming a good initial model ($\bar{\theta}_j$) the perturbations can be linearised

$$R_j(\rho_j,\theta_j) \approx R_j(\rho_j,\bar{\theta}_j) + \left[\nabla_\theta R\right]\delta\theta$$

and a “mosmod” correction made in terms of the perturbations $\delta\theta$. In the case of atmospheric correction with some available PIFs this is a very effective means of producing balanced mosaics in physical reflectance units with BRDF correction to standard sun and view angles.

2.5 “Forward” and “Inverse” Mappings

A “forward” mapping is FROM a Page TO the Base Page. An “inverse” map is FROM the Base Page TO a Page. The inverse map is obviously obtained by choosing the given Page as the Base Page and keeping the model relating the previous Base Page to the given (new Base) Page.

It will also help the discussion to relate the Mosmod style of geometric fitting back this more usual use of forward and inverse transformations used in image processing.

The roles of base page and overlaps and even reference Pages can be illustrated by considering the simple case of only three Pages. In the geometric case consider that there are three Pages A, B (eg two images) and M (eg a Map). The images and the map have control points, some of which are identified as GCPs (common to one of A and B and the Map) or Tie points (common to A and B).

The Mosmod approach resolves around choosing the "base Page". That is where all the geometric transformations map.

Case 1 Forward transformation of images to Map:

The base Page is the map M.

Transformations are:

FROM A TO M
FROM B TO M

Constraints are:

Fit to map GCPs in Page A are small;
Fit to map GCPs in Page B are small;
Differences between mapped Tie points in overlap between A and B are small in the Map coordinate system (eg metres).

In this case, the map GCPs form the "right hand sides" of the least squares equations. The result is that both the transformations (their coefficients) are available after the solution is obtained. So for either A or B you can go FROM pixel, line to Easting, Northing. The transformations from A and B to the Map will be “improved” by the constraint that they also agree in the overlap.

Case 2 Inverse Map to image A:

The base Page now is image A

Transformations are:

FROM B TO A
FROM M TO A

Constraints are:

Fit to image A GCPs in Page B are small;

Fit to image A GCPs in Page M are small;
Differences between mapped Tie points in overlap between B and M are small in the coordinate system of A (eg pixels).

Now it is really just the same. The ties between A and B are measured in the coordinate system of A as are those between B and M and M and A. The key is to see that the ties between M and B are being measured in the coordinate system of A. Then it is all symmetrical.

Out of this you get the two transformations. Often you do not keep the transformation FROM B TO A but you could and I have done many applications where these cross-transformations are very useful.

Again, this mapping should be “improved” by ensuring that the image B and the Map are well registered at the same time as the transformations from A to B and A to M are established.

Case 3 Inverse map to image B:

The base Page now is image B.

Transformations are:

FROM M TO B
FROM A TO B

Constraints are:

Fit to image B GCPs in Page A are small;
Fit to image B GCPs in Page M are small;
Differences between mapped Tie points in overlap between A and M are small in the coordinate system of B (eg pixels).

Now it all follows as above. Note that the ties between A and M are in the coordinate system of B. It is all balanced in this way.

Again, you may only keep the transformation from M to B.

Obviously, A and B are acting as “reference” images for the transformations between the other image and the map. For example, if image B were a SPOT panchromatic image and Image A were a Daedalus (DATM) image then the transformation between the Map and the SPOT image could use control outside the DATM run to get high accuracy and there could be many Tie points generated between the SPOT image and the DATM image. In the end the transformations between the DATM image and the map may be all that is wished.

This approach can be used with both a highly constrained approach such as polynomial fitting or with a highly flexible approach like triangulation.

In the case of the radiometric balancing which is the primary subject of this Task I claim the situation is just the same. In this case, however, we are generally only interested in the transformation from Image counts in each image to (say) reflectance. The base Page is a Page like a “Map” where units are reflectance and Map geometry is assumed. GCPs are points where reflectance is known or assumed known (Pseudo-Invariant Features or PIFs). The Hi-

Res BRDF could be seen as the means of achieving this balanced approach and making its benefits widely implemented.

2.6 Realising the code

In order to realise all of these examples it is important for a set of base procedures to be developed and a general means of defining models needs to be set up. The people involved should also look at the AMBRALS code and discuss the design in detail before implementing the ideas.

Norm Campbell already has an effective automatic Tie point generating system I believe. What is needed are the flexible model units and the Mosmod style of fitting methods. It is a case of doing it!

3. Appendix - The microBRIAN mosmod example.

3.1 Background

The basic situation for Mosmod involves creating a collection of Coordinate files which contain selected points from different “Pages” (i.e. images and/or a map). In microBRIAN, each “coordinate” or point has a unique number and identified points in different Pages are associated by having the same coordinate number.

Mosaicking involves setting up models FROM each Page TO another based on the coordinate files which preserves overlapping relationships with some or all of the overlapping Pages. The fitted models are then used to navigate the separate Pages for location of data or resampling. In the case of navigating an image, the model needed has the FROM Page as the image and the TO Page as the map. Then the transformation indicates the geometric coordinates of a pixel. In the case of resampling (with REMAPPER) the transformation needed has the FROM Page as the map and the TO Page as the image so that the pixel from which data are to be taken can be located for a specific map coordinate.

Suppose there are N+1 Pages. The first step in Mosmod is to select the Coordinate files and set up the Incidence Matrix *OverLap*:

$$Overlap = \left[n_{ij} \right]$$

where n_{ij} is the number of coordinates in common between Pages *i* and *j*. At the *same* time, the number of independent overlapping points ($\max_j$) with other images should be computed.

The overlap is then used to produce the distance matrix between the Pages. The distance is the simple adjacency distance and indicates the existence unconnected Pages as well as showing which Pages are `far' from others. In a very precise sense, this overlap distance is an indication of the degree to which control is transferred to the other Pages.

The next step is to choose a FROM Page and a TO Page and the order of the model to be fitted. The model will be formed to map FROM the coordinate system of the FROM Page TO the coordinate system of the TO Page. If the FROM Page index is *f*, the TO Page index is *t* and the order of the model to be fitted is *p* then the following checks must be carried out:

1. Check whether $n_{tf} \geq p$. If it is then the other check can be skipped.
2. Otherwise, check whether $d_{tf} < \infty$ and $\sum_{j \neq f} n_{fj} \geq p$. If so, then the model may be feasible and can be attempted.
3. Otherwise, the model is not possible to fit as planned and should not be tried until something is done about it.

The *N* Pages other than the TO Page can be rearranged (sorted) so that the FROM Page is number 1 and the others are in increasing distance from the FROM Page. If the system will not fit into the possible space, Pages can then be deleted starting back from *N*. This needs re-computing and checking in some cases.

The Pages can also be checked for model feasibility. That is, $\sum_{k \neq j} n_{jk} \geq p$ for each *j*.

If a Page model is not feasible, the order of the model should be reduced until it is or the Page should be removed. That is, the other Pages may have an order less than the one selected for the FROM and TO Pages.

3.2 Setting up the Sytem

Let the vector α_{jt} represent the *p* parameters of the model for the j'th Page for $j = 1, N$ mapping the *j*'th Page to the TO Page. That is, the model is taken to have the form:

$$z_t(x) = \sum_{k=1}^{p} \alpha_{jtk} \, \phi_k(x)$$

where x is general notation for a point in the j'th Page to which the model is attached and z_t is the value of the mapped point in the TO Page. In our models, there is one such equation for each coordinate of the geometry.

The complete linear system can now be written:

$$\begin{bmatrix} A_1 & 0 & . & . & 0 \\ 0 & A_2 & . & . & 0 \\ . & . & . & . & . \\ 0 & . & . & 0 & A_N \\ B_{12} & -C_{12} & 0 & . & 0 \\ B_{13} & 0 & -C_{13} & . & 0 \\ . & . & . & . & . \\ B_{1N} & 0 & . & 0 & -C_{1N} \\ 0 & B_{23} & -C_{23} & . & 0 \\ . & . & . & . & . \\ 0 & B_{2N} & . & 0 & -C_{2N} \\ . & . & . & . & . \\ . & . & . & . & -C_{N-2\,N} \\ 0 & . & . & B_{N-1\,N} & -C_{N-1\,N} \end{bmatrix} \begin{bmatrix} \alpha_{1t} \\ \alpha_{2t} \\ . \\ . \\ \alpha_{Nt} \end{bmatrix} \approx \begin{bmatrix} z_{1t} \\ z_{2t} \\ . \\ z_{Nt} \\ 0 \\ . \\ 0 \\ . \\ . \\ . \\ . \\ . \\ 0 \\ 0 \end{bmatrix}$$

Where, the matrices A, B and C are evaluations of the model basis functions at the relevant points. In this system of equations, the *N* sets:

$$A_j\ \alpha_{jt} \approx z_{jt}$$

are the systems leading to the solutions for each Page separately which would be obtained by the current program MODEL in microBRIAN. The MosMod development is to introduce the tie point interactions implied in the full system above.

The numbers of lines in the first *N* row blocks are $n_{1t}, n_{2t}, ..., n_{Nt}$. If any of these is zero, the block is taken as missing. This would occur if there were no points common between the image in question and the TO image. Similarly, there are $N(N-1)/2$ potential row blocks in the lower section of the system. The number of lines in each of these is n_{ij} where the index *i* runs from 1 to $N-1$ and the index *j* runs from $i+1$ to *N*. Again, if any pair have no overlapping points the block is taken as missing.

The block matrices represent the mapping of the coordinates of the Tie points into the coordinate system of the Base Page (or TO Page). That is:

$$B_{jk}\alpha_{jt} - C_{jk}\alpha_{kt} \approx 0$$

represents the equality of the points in the overlap between Pages j and k (k>j) in the coordinate system of the Base or TO Page measured by small residuals.

3.3 Solving by Least Squares

The normal equations (neglecting weights for now) are:

$$\begin{bmatrix} D_1 & -H_{12} & -H_{13} & . & -H_{1N} \\ -H_{12}^T & D_2 & -H_{23} & . & -H_{2N} \\ -H_{13}^T & -H_{23}^T & D_3 & . & . \\ . & . & . & . & -H_{N-1N} \\ -H_{1N}^T & -H_{2N}^T & . & -H_{N-1N}^T & D_N \end{bmatrix} \begin{bmatrix} \alpha_{1t} \\ \alpha_{2t} \\ . \\ . \\ \alpha_{Nt} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ b_N \end{bmatrix}$$

where:

$$D_j = A_j^T A_j + \sum_{k=j+1}^{N} B_{jk}^T B_{jk} + \sum_{k=1}^{j-1} C_{kj}^T C_{kj}$$

$$H_{ij} = B_{ij}^T C_{ij} \quad (j > i)$$

and

$$b_j = A_j^T z_{jt}$$

This system is easily solved and statistics generated using Block Cholesky. This is done in microBRIAN program Mosmod. Note that the same formulation can be used for image radiance balancing if the model is linear. This includes simple scaling of each image separately and kernel models. In the case of nonlinear models this solution will be applied to the Jacobian of the model in a Gauss iteration. Again, Block Cholesky is the best means of obtaining a solution.

3.4 Appendix to Appendix. Algorithm for Page Distances

The distance algorithm is as follows:

let

$$A = \left[a_{ij} \right]$$

be the Adjacency matrix for the set of Pages. That is,

$$a_{ii} = 0 \text{ for } i = 1, Npage$$
$$a_{ij} = 1 \text{ if } Ovrlap(i, j) > 0$$
$$a_{ij} = 0 \text{ if } Ovrlap(i, j) = 0 \text{ for } j \neq i$$

The distance matrix (*dist(i,j)*) records the minimum number of graph links that must be traversed to get from any Page *i* to Page *j*. This is the minimum power of the Adjacency matrix such that the (*i*,*j*) element becomes nonzero and leads to the following algorithm:

1. Set $n = 1$, $B = A$, $dist(ii) = 0$ and $dist(ij) = -1$ for $j \neq i$.
2. If, for $j \neq i$, $b_{ij} > 0$ and $dist(ij) < 0$, set $dist(ij) = n$ and set *modify=.true.*
3. If no elements were set (*modify=.false.*), all reachable Pages have been found so *Exit*, else, set:

$$B = B\,A\#$$
$$n = n + 1$$

(where A# indicates $b_{ij}\# = 1$ if $b_{ij} > 0$) and go to Step 2.

The resulting matrix *dist* has the required distances and where its value is -1 it indicates there is no connection between the Pages. In practice, only the strict upper triangle of *dist* needs to be stored and accessed.